

Quantum fields, cosmological constant and symmetry doubling

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Abstract

We explore how energy-parity, a protective symmetry for the cosmological constant [1], arises naturally in the classical phase space dynamics of matter. We derive and generalize the Liouville operator of electrodynamics, incorporating a “varying alpha” and diffusion. In this model, a one-parameter deformation connects classical ensemble and quantum field theory.

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1 Introduction

The cosmological constant problem is one of the outstanding unsolved problems of physics [2]. Besides approximate coincidence of its value with the average matter density of the universe in the present epoch, the problem consists in its smallness, $\Lambda \approx 10^{-123} M_{\text{Pl}}^4$. One would expect corrections on the order of typical particle physics scales, for example, induced by interactions of the Standard Model. No cancellation mechanism with the required finesse is known.

Addressing the smallness of Λ , the “energy-parity” symmetry introduced by Kaplan and Sundrum (KS) might protect the cosmological constant [1]. Schematically, it concerns the mapping $\text{Energy} \rightarrow -\text{Energy}$, where the energy includes the contributions of all charged matter and gauge fields (summarily called matter henceforth). Correspondingly, the following effective low-energy Lagrangian coupling gravity and matter is considered:

$$\mathcal{L} \equiv \sqrt{-g} \left(M_{\text{Pl}}^2 R - \Lambda_0 + \mathcal{L}_{\text{mat}}(\Phi) - \mathcal{L}_{\text{mat}}(\tilde{\Phi}) \right) , \quad (1)$$

where $g_{\mu\nu}$ denotes the metric entering the Einstein-Hilbert part of the Lagrangian, Λ_0 is the bare cosmological constant, and Φ stands for the set of all minimally coupled “visible” fields, while $\tilde{\Phi}$ denotes an identical “ghost” copy of the set of visible fields. The Lagrangian *function*, \mathcal{L}_{mat} , is the same for the visible and ghost sectors.

The main point of Lagrangian (1) is the relative sign between visible and ghost matter contributions. This leads to equal and opposite vacuum energies for both sectors. Therefore, they cancel and do not contribute to the bare and possibly small cosmological constant, Λ_0 . Such a scenario has been earlier proposed by Linde [3] (in a modified form in Ref. [4]). Similar ideas based on symmetries have more recently also been discussed in Refs. [5, 6, 7, 8]. However, vacuum instabilities due to visible-ghost couplings threaten all models with fields that contribute

negatively to the energy content of matter. KS have shown that vacuum decay may be acceptably slow under reasonable assumptions and is compatible with inflation and standard Big Bang cosmology [1].

Our aim presently is threefold:

- To show that the *energy-parity symmetry* arises, if dynamics encoded in $\mathcal{L}_{mat}(\Phi)$ is described in phase space. For this purpose, we consider a Hilbert space representation of the Liouville equation.
- To show that this *classical Liouville equation*, written in terms of appropriate variables, is related to the *functional Schrödinger equation*, i.e. quantum field theory, pertaining to the matter Lagrangian, $\mathcal{L}_{mat}(\Phi) - \mathcal{L}_{mat}(\tilde{\Phi})$. However, we will also find here destabilizing visible-ghost couplings.
- To show that the visible-ghost couplings are eliminated by incorporating *diffusion* into the Liouville operator, consistently with energy-parity and gauge symmetry, and by introducing a varying gauge coupling, similarly as in “varying alpha” or dilaton models [9, 10, 11, 12]. This results in a one-parameter interpolation between the *classical ensemble theory* and the *quantum field theory* (QFT) for the matter Lagrangian in Eq. (1).

Thus, one might speculate about a relation between cosmological constant problem, energy-parity symmetry, “varying alpha” models, and recently studied *deterministic dynamics beneath quantum theory*. – The latter investigations have been initiated by work of ’t Hooft, motivated by the conceptual problems in unifying general relativity and quantum theory [13, 14, 15].

There have always been arguments for and against the possibility to derive quantum theory from more fundamental and deterministic dynamical structures. The debate of hidden variables theories is well known. While much of this has come under experimental scrutiny, no deviation from quantum theory has been observed on the accessible scales. Nevertheless, it is quite plausible that quantum mechanics emerges as an effective theory only on sufficiently large scales compared to the Planck scale [13].

Our approach makes use of the remarkable similarity between Schrödinger and Liouville equation, when written in terms of suitable variables [16]. The Liouville operator (times i) is Hermitian in the operator approach to classical statistical mechanics. One would like to identify it with the *Hamiltonian of an emergent quantum system*. However, unlike the case of a quantum mechanical Hamilton operator, its spectrum is generally *not bounded from below*. Therefore, attempts to find a deterministic foundation of quantum theory by relating it to a classical ensemble theory, so far, had to face the difficult problem of constructing a stable ground state [13, 14, 16, 17, 18, 19, 20, 21].

The simplest emergent quantum models are based on a classical system evolving in discrete time steps (cellular automaton) [13, 18]. It appears that all classical Hamiltonian models turn into unitary quantum mechanical ones, if the Liouville operator is discretized [19]. The arbitrariness inherent in discretizations leaves enough freedom for the construction of a ground state. However, interacting field theories, so far, have resisted to this.

Various other arguments for deterministically induced quantum features have been presented – see works collected in Part III of Ref. [20], for example, or Refs. [22, 23], concerning statistical systems, quantum gravity, and matrix models. In detail, however, many of these incorporate variants of stochastic quantization procedures of Nelson and of Parisi and Wu based on an unknown mechanism driving the fluctuations [24].

Considering deterministic real-time evolution, we will show here that the characteristic doubling of classical phase space degrees of freedom, as compared to the quantum mechanical case,

gives rise to the visible and ghost sectors of the KS energy-parity scenario. In this way, the previous difficulty stemming from the presence of the negative energy sector is turned into a virtue. However, destabilizing visible-ghost couplings arise and need to be rendered harmless.

In Section 2, we begin with a classical $U(1)$ gauge theory, such as electrodynamics, with charged particles represented by complex Grassmann algebra valued fields [16]. We admit a variable gauge coupling as in the “variable alpha” or dilaton models [9, 10, 11]. From Hamilton’s equations we obtain the Liouville equation. – We develop the equivalent Hilbert space formulation, which automatically incorporates a ghost copy of the visible sector. This *classical* equation resembles a functional Schrödinger equation in which visible and ghost fields contribute with opposite sign to the emergent Hamiltonian.

In Section 3, we introduce a diffusion term which is compatible with gauge invariance and energy-parity symmetry. It uniquely incorporates only one extra derivative in the field variables without additional dimensionfull parameters. – Depending on the variable coupling, considered as a deformation parameter here, the *extended equation interpolates between a classical phase space ensemble theory and the functional Schrödinger equation*. In the latter limit, visible-ghost matter couplings are absent and KS energy-parity symmetry is manifest.

In Section 4, we briefly summarize and point out interesting topics for future study.

2 The Liouville operator equation for a classical gauge theory

A charged matter field can be described by “pseudoclassical mechanics”. This has been introduced through work of Casalbuoni and of Berezin and Marinov, who considered a *Grassmann variant of classical mechanics*, studying the dynamics of spin degrees of freedom classically and after quantization as usual [25]. For some recent applications, see Refs. [16, 26], for example.

Thus, we introduce the complex four-component spinor field, ψ , which takes its “fermionic” character from the generators of an infinite dimensional Grassmann algebra [26]. They obey:

$$\{\psi(x), \psi(x')\}_+ \equiv \psi(x)\psi(x') + \psi(x')\psi(x) = 0 \quad , \quad (2)$$

where x, x' are equal-time coordinate labels and spinor indices are suppressed. – Furthermore, the usual four-vector potential, A_μ , $\mu = 0, 1, 2, 3$, defines the gauge field, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. – We assume the Minkowski metric $g_{\mu\nu} \equiv \text{diag}(+1, -1, -1, -1)$, since the essential aspects in the following do not depend on the background metric.

Then, the *classical $U(1)$ gauge theory* to be studied is defined by the action:

$$S \equiv \int d^4x \mathcal{L}_{mat}(\psi, A) \quad (3)$$

$$\equiv \int d^4x \left(\bar{\psi}(i\gamma \cdot D - m)\psi - \frac{1}{4\epsilon^2} F^2 \right) \quad , \quad (4)$$

where $\bar{\psi}_b \equiv \psi_a^* \gamma_{ab}^0$ (γ ’s denoting the Dirac gamma matrices and $a, b = 1, \dots, 4$ spinor indices), m is the mass parameter, and the covariant derivative is defined by $D_\mu \equiv \partial_\mu + ie_0 A_\mu$. We also introduced here the scalar, dimensionless, and gauge neutral field ϵ , such that the electric charge is given by $e(x) = e_0 \epsilon(x^\mu)$ [9, 11]. Thus, the field ϵ^{-1} plays the role of a dielectric function of the vacuum and, generally, varies in space and time. – Models with varying coupling constants (or dilaton fields) have originated in various contexts [9, 10, 11, 12]. For our purposes, it is sufficient to consider ϵ as a variable *deformation parameter* of our example gauge theory.

Proceeding as usual, we calculate the fermionic canonical momentum ($\int d^4x \mathcal{L}_{mat} \equiv \int dt L$):

$$\Pi \equiv -\frac{\delta L}{\delta \partial_0 \psi} = i\psi^* \quad , \quad (5)$$

with a functional left derivative here. Concerning Grassmann variables, we will always use *left derivatives* [26]; for example, $\delta_\psi \equiv \delta_\psi^L$, with:

$$\delta_\psi^L(x)\psi(y)\psi(z) \equiv \delta^3(x-y)\psi(z) - \psi(y)\delta^3(x-z) . \quad (6)$$

The momenta conjugate to the gauge potentials are:

$$\Pi_0 \equiv \frac{\delta L}{\delta \partial_0 A^0} = 0 , \quad (7)$$

$$\Pi_i \equiv \frac{\delta L}{\delta \partial_0 A^i} = -\epsilon^{-2} F_{0i} \equiv \epsilon^{-2} E^i . \quad (8)$$

Then, we obtain the Hamiltonian:

$$\begin{aligned} H &= \int d^3x \left(\Pi \partial_0 \psi + \Pi_i \partial_0 A^i \right) - L \\ &= \int d^3x \left(-\Pi \gamma^0 (\gamma_j D^j + im) \psi + \frac{\epsilon^2}{2} \Pi_i \Pi_i + \frac{1}{4\epsilon^2} F_{ij} F^{ij} + A^0 (\partial_i \Pi_i - ie_0 \Pi \psi) \right) , \end{aligned} \quad (9)$$

after partially integrating the penultimate term, and summing over pairwise equal indices always.

It is obvious from Eq. (7) and the term involving A^0 in Eq. (9) that A^0 is the Lagrange multiplier which incorporates Gauss' law as a constraint:

$$G \equiv -\frac{\delta L}{\delta A^0} = \partial_i \Pi_i - ie_0 \Pi \psi = 0 . \quad (10)$$

Keeping this constraint in mind, we work in temporal axial gauge, $A^0 \equiv 0$, from now on.

For the phase space description of the classical field theory, it will be useful to introduce the Poisson bracket operation, acting on two observables O_1 and O_2 which are function(al)s of the phase space variables Π, ψ, Π_i, A^i and may explicitly depend on time:

$$\{O_1, O_2\} \equiv \int d^3x \left(\frac{\delta O_1}{\delta \Pi} \frac{\delta O_2}{\delta \psi} + \frac{\delta O_1}{\delta \psi} \frac{\delta O_2}{\delta \Pi} + \frac{\delta O_1}{\delta \Pi_i} \frac{\delta O_2}{\delta A^i} - \frac{\delta O_1}{\delta A^i} \frac{\delta O_2}{\delta \Pi_i} \right) , \quad (11)$$

where all functional (left) derivatives refer to the same space-time argument. This Poisson bracket is *graded* antisymmetric: it is antisymmetric, if both observables O_1, O_2 are Grassmann even, it is symmetric, if both are odd, while in the remaining cases the terms involving Grassmann derivatives contribute symmetrically and the others antisymmetrically [25, 26].

For any observable O , the usual relation among time derivatives holds:

$$\frac{d}{dx^0} O = \{H, O\} + \partial_0 O , \quad (12)$$

which embodies Hamilton's equations of motion. For example, $d\psi/dx^0 = \{H, \psi\}$ and $d\Pi/dx^0 = \{H, \Pi\}$, i.e. the Dirac equation and its adjoint. The time independent Hamiltonian of Eq. (9) is conserved. One also verifies that $\{H, G\} = 0$, expressing the gauge invariance of the evolution of the system. Consequently, it is sufficient to implement Gauss' law, Eq. (10), at one time. – We now turn to the study of an ensemble of systems that are described by Eq. (4).

2.1 The Hilbert space representation

A particular example of Eq. (12) is the Liouville equation for a conservative system. Considering an ensemble, especially with some distribution over initial conditions, this equation governs the evolution of its phase space density ρ :

$$0 = \frac{d}{dx^0} \rho = \partial_0 \rho - \hat{\mathcal{L}} \rho , \quad (13)$$

$$- \hat{\mathcal{L}} \rho \equiv \{H, \rho\} , \quad (14)$$

where $\hat{\mathcal{L}}$ is the Liouville operator. These equations summarize the classical statistical mechanics of a conservative system, given the Hamiltonian H .

An equivalent Hilbert space formulation is obtained in the operator approach, with appropriate modifications for our classical field theory, which is based on two ingredients:

1. The phase space density functional can be factorized in the form $\rho \equiv \Psi^* \Psi$. This surprising fact is guaranteed by a theorem proven by 't Hooft recently [15].
2. The Grassmann algebra valued complex state functional Ψ itself obeys Eq. (13).

Furthermore, the complex valued inner product of such state functionals is defined by:

$$\langle \Psi | \Phi \rangle \equiv \int \mathcal{D}\Pi \mathcal{D}\psi \mathcal{D}\Pi_j \mathcal{D}A^i \Psi^* \Phi = \langle \Phi | \Psi \rangle^* , \quad (15)$$

functionally integrating over all phase space variables (fields). – Due to the presence of Grassmann variables, the $*$ -operation which defines the dual of a state functional needs special attention. It amounts to complex conjugation for a “bosonic” state functional, $(\Psi[\Pi_j, A^i])^* \equiv \Psi^*[\Pi_j, A^i]$, analogously to an ordinary wave function in quantum mechanics.

However, based on complex conjugation alone, the inner product involving Grassmann variables would not be well defined, in particular, it would not necessarily yield a complex number nor a positive definite norm. Instead, a detailed construction of the inner product for functionals of Grassmann algebra valued fields has been carried out in Ref. [27], which has these physically motivated properties. Other constructions are possible [28]; see also the discussion in the Appendix of Ref. [27]. Further applications can be found in Refs. [29].

Thus, the notion of square-integrable functions can be extended to the phase space functionals (rigorously after discretization). Furthermore, only “physical” state functionals which conform with Gauss’ law (cf. below) are admitted here.

Given the Hilbert space structure, the operator $\hat{\mathcal{H}} \equiv i\hat{\mathcal{L}}$ has to be Hermitian for a conservative system and the positive overlap $\langle \Psi | \Psi \rangle$ is a conserved quantity. Then, the Liouville equation also applies to $\rho = \Psi^* \Psi$, due to its linearity. Interpreting ρ as the phase space distribution, its moments yield the physically meaningful expectation values of observables, as usual.

The similarity with the usual quantum mechanical formalism is striking. In order to expose this more clearly, we further transform the functional equation implied by the above two postulates together with Eqs. (13)–(14). Let us write this equation in the suggestive form:

$$i\partial_t \Psi = \hat{\mathcal{H}} \Psi , \quad (16)$$

where Ψ is a functional of Π, ψ, Π_i, A^i , and where the effective “Hamilton operator” is:

$$\hat{\mathcal{H}} \Psi = -i\{H, \Psi\} \equiv (\hat{\mathcal{H}}_\psi + \hat{\mathcal{H}}_A) \Psi . \quad (17)$$

The operators $\hat{\mathcal{H}}_\psi$ and $\hat{\mathcal{H}}_A$, respectively, refer to terms which originate from the Poisson bracket either involving Grassmann derivatives or not, see Eq. (11). We consider both terms in turn.

2.2 The gauge field operator $\hat{\mathcal{H}}_A$

Beginning with the gauge field part, we obtain:

$$\hat{\mathcal{H}}_A \Psi = -i \int d^3x \left(\epsilon^2 \Pi_i \frac{\delta}{\delta A^i} - \left[\frac{1}{\epsilon^2} (\partial^j F_{ij}) - i e_0 \Pi \gamma^0 \gamma_i \psi \right] \frac{\delta}{\delta \Pi_i} \right) \Psi . \quad (18)$$

The form of the kinetic term suggests to perform a functional Fourier transformation:

$$\Psi[\Pi_i] = \int \mathcal{D}A^{i\prime} \exp(-i \Pi_i \cdot A^{i\prime}) \Psi[A^{i\prime}] , \quad (19)$$

where all other variables are momentarily suppressed; in the exponent, an integration over space is understood. In the transformed variables, the Eq. (17) reads:

$$\hat{\mathcal{H}}_A \Psi = \int d^3x \left(-\epsilon^2 \frac{\delta}{\delta A^i} \frac{\delta}{\delta A^{i\prime}} + \frac{1}{2\epsilon^2} F_{ij} F^{ij\prime} - i e_0 \Pi \gamma^0 \gamma_i \psi A^{i\prime} \right) \Psi , \quad (20)$$

making use of suitable partial integrations, and where $F^{ij\prime} \equiv \partial^i A^{j\prime} - \partial^j A^{i\prime}$. Next, we perform a linear transformation of the gauge field variables:

$$A^i \equiv \frac{1}{\sqrt{2}}(a^i + \tilde{a}^i) , \quad A^{i\prime} \equiv \frac{1}{\sqrt{2}}(a^i - \tilde{a}^i) . \quad (21)$$

This transforms the operator of Eq. (20) into:

$$\hat{\mathcal{H}}_A = \int d^3x \left(-\frac{\epsilon^2}{2} \left(\frac{\delta}{\delta a^i} \frac{\delta}{\delta a^i} - \frac{\delta}{\delta \tilde{a}^i} \frac{\delta}{\delta \tilde{a}^i} \right) + \frac{1}{4\epsilon^2} (F_{ij} F^{ij} - \tilde{F}_{ij} \tilde{F}^{ij}) - i e'_0 \Pi \gamma^0 \gamma_i \psi (a^i - \tilde{a}^i) \right) , \quad (22)$$

where $e'_0 \equiv e_0/\sqrt{2}$, $F^{ij} \equiv \partial^i a^j - \partial^j a^i$, and $\tilde{F}^{ij} \equiv \partial^i \tilde{a}^j - \partial^j \tilde{a}^i$. – Thus, we find that a ghost copy in terms of \tilde{a} arises here with opposite sign for each visible gauge field term involving a .

2.3 The fermionic field operator $\hat{\mathcal{H}}_\psi$

Proceeding with the fermionic field part, we obtain:

$$\hat{\mathcal{H}}_\psi \Psi = \int d^3x \left(-\psi [\gamma^0 (-i \gamma_j \overleftarrow{D}^{j\prime} + m)]^t \delta_\psi + \Pi \gamma^0 (-i \gamma_j \overrightarrow{D}^{j\prime} + m) \delta_\Pi \right) \Psi , \quad (23)$$

indicating which way the derivatives act, with $D^{j\prime} \equiv \partial^j + i e'_0 (a^j + \tilde{a}^j)$, and where $[\dots]^t$ denotes spinor matrix transposition. Explicitly, the first term is: $\psi M^t \delta_\psi \equiv \psi_b (M^t)_{ba} \delta_{\psi_a} = M_{ab} \psi_b \delta_{\psi_a}$.

Making use of the algebra of γ -matrices – in particular, the *charge conjugation* matrix C and the matrix γ_5 , with $C \gamma_\mu C^{-1} = -[\gamma_\mu]^t$, $C^2 = -I$, $\{\gamma_5, \gamma_\mu\}_+ = 0$, and $\gamma_5^2 = I$ [30] – and of a partial integration in the first term, we rewrite the operator $\hat{\mathcal{H}}_\psi$ as:

$$\hat{\mathcal{H}}_\psi = \int d^3x \left(-\psi_C h_D^- \delta_{\psi_C} + \Pi h_D^+ \delta_\Pi \right) , \quad (24)$$

with:

$$\psi_C \equiv \psi (\gamma_5 C)^{-1} , \quad (25)$$

$$h_D \equiv \gamma^0 (-i \gamma_j D^{j\prime} + m) , \quad (26)$$

i.e., the Hermitian kernel of the Dirac Hamiltonian, with the superscript “ \pm ” indicating the sign of the minimal coupling term in the covariant derivative.

We may go one step further, employing the *time reversal* matrix T [30]. It is characterized by the relation $T\gamma_\mu T^{-1} = \gamma^\mu$ and involves complex conjugation, such that $T(i\gamma_\mu)T^{-1} = -i\gamma^\mu$, for example. Thus, the sign of the coupling in the first term in Eq. (24) can be changed: $Th_D^- T^{-1} = h_D^+$. Correspondingly, we introduce:

$$\psi_{CT} \equiv \psi_C T^{-1} = \psi(T\gamma_5 C)^{-1} , \quad (27)$$

and obtain:

$$\hat{\mathcal{H}}_\psi = \int d^3x \left(-\psi_{CT} h_D \delta_{\psi_{CT}} + \Pi h_D \delta_\Pi \right) , \quad (28)$$

with $h_D = h_D^+$, from now on.

As suggested by Eqs. (22) and (28), we interpret Π as ghost copy of ψ_{CT} . Summarizing our findings, we state the *energy-parity symmetry* transformations:

$$\psi_{CT} \longleftrightarrow \Pi , \quad (29)$$

$$a_i \longleftrightarrow \tilde{a}_i . \quad (30)$$

Applying these to the effective Hamiltonian operator, $\hat{\mathcal{H}} = \hat{\mathcal{H}}_\psi + \hat{\mathcal{H}}_A$, we obtain indeed:

$$\hat{\mathcal{H}} \longleftrightarrow -\hat{\mathcal{H}} . \quad (31)$$

A little algebra is needed, in order to show that the interaction term $\propto e'_0$ in $\hat{\mathcal{H}}_A$, see Eq. (22), conforms with (31); in particular, $[T\gamma_5 C]^t = -T\gamma_5 C$.

This result demonstrates that the energy-parity symmetry which has been postulated by Kaplan and Sundrum [1] arises naturally in the Hilbert space representation of the classical ensemble theory for our U(1) model of Eq. (3).

2.4 Gauge invariance

Let us briefly discuss the operator version of Gauss' law, Eq. (10). Similarly as the effective Hamiltonian in the previous subsections, we obtain the *Gauss' law operator*:

$$\hat{\mathcal{G}}\Psi \equiv -i\{G, \Psi\}/\sqrt{2} \quad (32)$$

$$= \left(\frac{i}{2} \partial_i (\delta_{a^i} + \delta_{\tilde{a}^i}) - e'_0 \psi \delta_\psi + e'_0 \Pi \delta_\Pi \right) \Psi , \quad (33)$$

with an extra factor of $\sqrt{2}$ for later convenience. Locally, we then have the Gauss' law constraint:

$$\hat{\mathcal{G}}(x)\Psi = 0 , \quad (34)$$

which needs to be implemented together with the implicit gauge condition $A^0 = 0$, in order to eliminate unphysical gauge degrees of freedom. – Finally, a straightforward calculation yields the commutator:

$$[\hat{\mathcal{H}}, \hat{\mathcal{G}}(x)]\Psi = 0 , \quad (35)$$

which, again, expresses the local U(1) gauge invariance of the evolution generated by $\hat{\mathcal{H}}$. Therefore, the constraint can be implemented consistently as an initial condition, for example.

This completes the derivation of classical statistical mechanics for a U(1) gauge field theory in a Hilbert space formalism, uncovering the intrinsic energy-parity symmetry.

3 “Varying alpha”, diffusion and transition to QFT

It will be shown here that the classical ensemble theory of Section 2 presents a limit of a more general ensemble theory. In another limit, varying the deformation parameter ϵ , this reduces to a quantized gauge field theory in the Schrödinger picture.

We reparametrize ϵ , as introduced in the action, Eq. (3), for variable coupling [9, 10]:

$$\epsilon^2 \equiv \epsilon^2(\epsilon') \equiv \frac{1}{2}\epsilon'(1 + \epsilon') \quad . \quad (36)$$

Furthermore, the following linear transformation of the vector fields is implemented:

$$a + \tilde{a} \equiv \frac{1}{2}(1 + \epsilon')(a' + \tilde{a}') \quad , \quad (37)$$

$$a - \tilde{a} \equiv \epsilon'(a' - \tilde{a}') \quad . \quad (38)$$

Contributions to the effective Hamilton operator, $\hat{\mathcal{H}} = \hat{\mathcal{H}}_\psi + \hat{\mathcal{H}}_A$, previously obtained in Eqs. (22) and (23), respectively, are transformed accordingly. We obtain:

$$\begin{aligned} \hat{\mathcal{H}}_A = & \int d^3x \left(-\frac{1}{2} \left(\frac{\delta}{\delta a^i} \frac{\delta}{\delta a^i} - \frac{\delta}{\delta \tilde{a}^i} \frac{\delta}{\delta \tilde{a}^i} \right) + \frac{1}{4} (F_{ij}(\epsilon') F^{ij}(1 + \epsilon') - \tilde{F}_{ij}(\epsilon') \tilde{F}^{ij}(1 + \epsilon')) \right. \\ & \left. + \frac{1}{4(1 + \epsilon')} (F_{ij}(\epsilon') \tilde{F}^{ij}(1) - F_{ij}(1) \tilde{F}^{ij}(\epsilon')) - i e'_0 \epsilon' \Pi \gamma^0 \gamma^i \psi (a'_i - \tilde{a}'_i) \right) \quad , \end{aligned} \quad (39)$$

with $F^{ij}(\xi) \equiv \xi^{-1}(\partial^i \xi a^j - \partial^j \xi a^i)$, and $\tilde{F}^{ij}(\xi) \equiv \xi^{-1}(\partial^i \xi \tilde{a}^j - \partial^j \xi \tilde{a}^i)$; the latter present the appropriate generalization of the usual field strength tensor [9, 10, 11], which is recovered in the case of spatially constant ξ . – Finally, the covariant derivative transforms to:

$$D'^j \equiv \partial^j + i \frac{e'_0}{2} (1 + \epsilon') (a'^j + \tilde{a}'^j) \quad , \quad (40)$$

and, with this, the fermionic contribution, $\hat{\mathcal{H}}_\psi$, *retains its form*, as before in Eqs. (23) or (28).

The varying *effective charge* $e_{eff} \equiv e'_0(1 + \epsilon')/2$, which is seen in Eq. (40), also enters the correspondingly transformed Gauss’ law operator from Eq. (33):

$$(e'_0)^{-1} \hat{\mathcal{G}} = \frac{i}{2} \partial_i e_{eff}^{-1} (\delta_{a^i} + \delta_{\tilde{a}^i}) + \psi_{CT} \delta_{\psi_{CT}} + \Pi \delta_\Pi \quad . \quad (41)$$

Note that $\psi i \delta_\psi = -\psi_{CT} \delta_{\psi_{CT}}$, cf. Eqs. (10), (27). – So far, this is still the Hilbert space version of statistical mechanics and Liouville equation, in particular, for our classical field theory.

3.1 Generalization of the Liouville equation

We now incorporate an *additional interaction* term, $\hat{\mathcal{H}}_{int}$, into the effective Hamilton operator:

$$\hat{\mathcal{H}} \equiv \hat{\mathcal{H}}_\psi + \hat{\mathcal{H}}_A + \hat{\mathcal{H}}_{int} \quad , \quad (42)$$

$$\hat{\mathcal{H}}_{int} \equiv - \int d^3x \left(\frac{e'_0}{2} (1 - \epsilon') (a'^j - \tilde{a}'^j) (\psi [\gamma^0 \gamma_j]^t \delta_\psi + \Pi \gamma^0 \gamma_j \delta_\Pi) \right) \quad . \quad (43)$$

This should be compared to the minimal coupling terms in $\hat{\mathcal{H}}_\psi$, Eq. (23), inserting the covariant derivative of Eq. (40); using $\psi [\gamma^0 \gamma_j]^t \delta_\psi = \psi_{CT} \gamma^0 \gamma_j \delta_{\psi_{CT}}$, yields the appropriate term for comparison with Eq. (28). – Since $\hat{\mathcal{H}} \equiv i \hat{\mathcal{L}}$, this generalizes the classical ensemble theory.

Several remarks are in order here characterizing the new term $\hat{\mathcal{H}}_{int}$:

- It vanishes in the limit $\epsilon' \rightarrow 1$, where Eq. (42) presents the Hilbert space version of the Liouville operator (times i) of the classical theory with gauge coupling constant e_0 .
- It involves one extra phase space derivative as compared to all other terms in $\hat{\mathcal{H}}$, which were generated by the Poisson bracket, Eq. (11), without introducing an additional dimensionfull parameter. Recall that $a'^j - \tilde{a}'^j \propto i\delta/\delta\Pi_j$, by Fourier transformation, where Π_j is the original canonical momentum variable conjugate to A^j .
- Since it is Hermitian (cf. below), it follows that $\hat{\mathcal{H}}_{int}$ represents a *diffusive interaction*.
- It is gauge invariant, since $[\hat{\mathcal{H}}_{int}, \hat{\mathcal{G}}] = 0$, and conforms with energy-parity, Eqs. (29)–(31).

3.2 Unitarity

In order that the time evolution of the system, described by the functional Ψ , be unitary, the Hamilton operator $\hat{\mathcal{H}}$ needs to be Hermitian.

The pure gauge field part of $\hat{\mathcal{H}}_A$, Eq. (39), fullfills this. It represents the Hamilton operator of two interacting *quantized gauge fields*, a_i and ghost copy \tilde{a}_i , in the Schrödinger picture. Their interaction is solely due to the *spatial variation* of the coupling, since the terms $\propto (1 + \epsilon')^{-1}$ in Eq. (39) cancel for constant ϵ' . – The construction of the corresponding function space, on which these bosonic operators act, follows the scalar field case reviewed in Ref. [27].

Turning to the Grassmann variables, we now consider the field ψ_{CT} and the functional derivative $\delta\psi_{CT}$ as a representation of a *charged fermion field operator* $\hat{\psi}$ and its adjoint $\hat{\psi}^\dagger$, respectively. This is suggested by the equal-time anticommutation relations:

$$\{\psi(x), \frac{\delta}{\delta\psi(x')}\}_+ = \delta^3(x - x') = \{\hat{\psi}(x), \hat{\psi}^\dagger(x')\}_+ , \quad (44)$$

suppressing spinor indices, and analogously for $\Pi, \delta\Pi$ and $\hat{\Pi}, \hat{\Pi}^\dagger$. Symbolically, we relate:

$$(\psi_{CT}; \delta\psi_{CT}) \longleftrightarrow (\hat{\psi}; \hat{\psi}^\dagger) , \quad (45)$$

$$(\Pi; \delta\Pi) \longleftrightarrow (\hat{\Pi}; \hat{\Pi}^\dagger) . \quad (46)$$

Thus, the subscript “... $_{CT}$ ” is absorbed in the overhead “ $\hat{\cdot}$ ” in what follows.

Let us specify in more detail the space of functionals, on which the fermionic field operators act. Decomposing the fields ψ, Π into real and imaginary parts:

$$\psi \equiv \frac{1}{\sqrt{2}}(\psi_R + i\psi_I) , \quad \Pi \equiv \frac{1}{\sqrt{2}}(\Pi_R + i\Pi_I) , \quad (47)$$

we associate operators with the real components:

$$\hat{\psi}_R \equiv \frac{1}{\sqrt{2}}(u + \delta_u) , \quad \hat{\Pi}_R \equiv \frac{1}{\sqrt{2}}(\tilde{u} + \delta_{\tilde{u}}) , \quad (48)$$

$$\hat{\psi}_I \equiv \frac{1}{i\sqrt{2}}(u - \delta_u) , \quad \hat{\Pi}_I \equiv \frac{1}{i\sqrt{2}}(\tilde{u} - \delta_{\tilde{u}}) , \quad (49)$$

where u, \tilde{u} are real (four-component) Grassmann fields.

These operators act on functionals $\Psi[u, \tilde{u}]$. Constructing the dual $\Psi^*[u, \tilde{u}]$ as in Refs. [27, 28], the adjoints of u and \tilde{u} are δ_u and $\delta_{\tilde{u}}$, respectively. Therefore, the operators $\hat{\psi}_{R,I}, \hat{\Pi}_{R,I}$ are

Hermitian here. It follows that $\hat{\psi} = u$ and $\hat{\psi}^\dagger = \delta_u$, as well as $\hat{\Pi} = \tilde{u}$ and $\hat{\Pi}^\dagger = \delta_{\tilde{u}}$. In this way, the anticommutator relations (44) are realized, and analogous ones for $\hat{\Pi}, \hat{\Pi}^\dagger$.

We adopt the convention of Berezin and Marinov that the *adjoint of a product* of Grassmann variables ξ_1, ξ_2 , incorporating complex conjugation, is: $(\xi_1 \xi_2)^\dagger = \xi_2^\dagger \xi_1^\dagger$ [25]. With this, the space of functionals, and the above realizations of the fields as operators, we have:

$$(\hat{\mathcal{H}}_\psi + \hat{\mathcal{H}}_{int})^\dagger = \hat{\mathcal{H}}_\psi + \hat{\mathcal{H}}_{int} , \quad (50)$$

$$\hat{\mathcal{G}}^\dagger = \hat{\mathcal{G}} . \quad (51)$$

see Eqs. (28), (43), and (41). Of course, the functionals $\Psi[u, a_i; \tilde{u}, \tilde{a}_i]$, generally, depend on all bosonic and fermionic fields, interpreted as visible (u, a_i) and ghost matter (\tilde{u}, \tilde{a}_i).

One term of $\hat{\mathcal{H}}$ is left to be considered. Coupling fermions and bosons, see $\hat{\mathcal{H}}_A$ in Eq. (39):

$$\hat{\mathcal{H}}_{A\psi\Pi} \equiv -ie'_0 \epsilon' \Pi \gamma^0 \gamma^i \psi (a'_i - \tilde{a}'_i) , \quad (52)$$

this, in general, is *not* Hermitian. Instead, the relevant factor of this term gives:

$$(i\hat{\Pi}\gamma^0\gamma^iT\gamma_5C\hat{\psi})^\dagger = -i(T\gamma_5C\hat{\psi})^\dagger\gamma^0\gamma^i\hat{\Pi}^\dagger \neq i\hat{\Pi}\gamma^0\gamma^iT\gamma_5C\hat{\psi} , \quad (53)$$

where we used Eq. (27) and (45), (46).

There are several ways to handle this situation, with different physical implications. – First, by adding $\hat{\mathcal{H}}_{A\psi\Pi}^\dagger$ to $\hat{\mathcal{H}}$, we can make the resulting Hamilton operator Hermitian, maintaining gauge invariance and energy-parity symmetry. In terms of the original phase space variables, this term incorporates three functional derivatives. It corresponds to a generalization of the Liouville operator, similarly as adding $\hat{\mathcal{H}}_{int}$, Eq. (43). While the latter is necessary for a smooth transition to quantum theory, as we shall see, we have no particular reason for the former addition. – Second, we can impose a constraint:

$$\hat{\mathcal{C}}_1\Psi \equiv (\hat{\Pi} - i(T\gamma_5C\hat{\psi})^\dagger)\Psi = 0 , \quad (54)$$

or a less restrictive constraint:

$$\hat{\mathcal{C}}_2\Psi \equiv (\hat{\mathcal{H}}_{A\psi\Pi} - \hat{\mathcal{H}}_{A\psi\Pi}^\dagger)\Psi = 0 , \quad (55)$$

and thereby reduce the Hilbert space, eliminating states that give rise to the anti-Hermitian part of $\hat{\mathcal{H}}_{A\psi\Pi}$. – The constraint $\hat{\mathcal{C}}_1$ reminds us of the classical Eq. (5). It eliminates ghost fermions as an independent field. In fact, there is a realization of the Hilbert space operators, differing from Eqs. (45)–(49), which automatically incorporates this constraint. Since it correlates ghost with visible fermions, even in the absence of interactions, it is not usefull here. The constraint $\hat{\mathcal{C}}_2$ eliminates only states that allow certain transitions between visible and ghost matter, in an ad hoc way. Since $[\hat{\mathcal{H}}, \hat{\mathcal{C}}_{1,2}] \neq 0$, neither constraint can be simply imposed on initial conditions. – Third, much less restrictive is the constraint:

$$\langle \Psi | \hat{\mathcal{C}}_{1,2} | \Psi \rangle = 0 , \quad (56)$$

which allows fluctuations away from the strict constraints, Eqs. (54), (55). In this case, the evolution of the system is not unitary. In particular, in the limit $\epsilon' \rightarrow 1$, which concerns the classical ensemble theory, the larger Hilbert space here contains states which experience dissipative forces. This is interesting in view of the “information loss” ideas of Refs. [13, 17, 21], which we mentioned in Section 1, and we will come back to it.

Finally, however, we observe that the non-Hermitian term $\hat{\mathcal{H}}_{A\psi\Pi}$ vanishes in the limit $\epsilon' \rightarrow 0$. This limit, in particular, will be further studied in what follows.

3.3 Symmetry doubling, energy-parity and quantum fields

Considering the *varying gauge coupling* induced by ϵ , as introduced in Eq. (4) and reparametrized in terms of ϵ' in Eq. (36), the underlying mechanism is *not* our present concern. It should be a low-energy reflection of a more fundamental theory than described by Eq. (1) and is amply discussed in the literature [9, 10, 11, 12].

However, the reader not wishing to adopt such an idea is invited to take the following as remarks on a *one-parameter deformation* of the classical ensemble or quantum field theory, respectively, for our U(1) model. This possibility in itself seems interesting.

As we have discussed already the “pseudoclassical” limit $\epsilon' \rightarrow 1$, we now study in more detail the limit $\epsilon' \rightarrow 0$. In this case, assuming that ϵ' is spatially homogeneous and collecting terms from Eqs. (28), (39), (40), (43), according to Eq. (42) and (45), (46), we obtain:

$$\hat{\mathcal{H}} = \int d^3x \left(\hat{\psi}^\dagger H_D \hat{\psi} - \frac{1}{2} \frac{\delta}{\delta a^i} \frac{\delta}{\delta \tilde{a}^i} + \frac{1}{4} F_{ij} F^{ij} - (\hat{\Pi}^\dagger \tilde{H}_D \hat{\Pi} - \frac{1}{2} \frac{\delta}{\delta \tilde{a}^i} \frac{\delta}{\delta a^i} + \frac{1}{4} \tilde{F}_{ij} \tilde{F}^{ij}) \right), \quad (57)$$

with the kernel of the Dirac Hamiltonian:

$$H_D \equiv \gamma^0 (-i\gamma_j (\partial^j + ie'_0 a^j) + m), \quad (58)$$

and where \tilde{H}_D has a^j replaced by \tilde{a}^j . This is the *Hamilton operator* of an U(1) gauge theory (fields ψ, a_i) in the Schrödinger picture, in temporal axial gauge, together with an identical ghost copy (fields Π, \tilde{a}_i). – Thus, we have derived the quantized matter part of the KS model of Eq. (1) for Abelian gauge symmetry [1].

While the usual interaction remains (coupling e'_0), destabilizing visible-ghost matter couplings are absent in this “quantum limit”. Of course, it is related to the particular additional interaction, $\hat{\mathcal{H}}_{int}$, introduced in Eq. (43). Further, note that the Gauss’ law operator of Eq. (41) decomposes into visible and ghost matter parts:

$$\hat{\mathcal{G}} = i\partial_i \delta_{a^i} + e'_0 \hat{\psi} \hat{\psi}^\dagger + i\partial_i \delta_{\tilde{a}^i} + e'_0 \hat{\Pi} \hat{\Pi}^\dagger \equiv \hat{\mathcal{G}}_{vis} + \hat{\mathcal{G}}_{gho}, \quad (59)$$

which, in the present limit, obey:

$$[\hat{\mathcal{H}}, \hat{\mathcal{G}}_{vis}] = [\hat{\mathcal{H}}, \hat{\mathcal{G}}_{gho}] = 0. \quad (60)$$

Thus, we find here the *doubled symmetry* $U(1)_{vis} \times U(1)_{gho}$, in agreement with the KS scenario.

Furthermore, the quantum limit has the following features seen in Eq. (4) or Eqs. (36)–(38), where correspondingly $\epsilon' \rightarrow 0$ and $\epsilon^2(\epsilon') \rightarrow 0$. Undoing the linear and Fourier transformations involved, the gauge field (operators) are related to the original phase space variables:

$$a'_j \sim \frac{i}{\epsilon'(\epsilon)} \delta_{\Pi j} + \frac{1}{1 + \epsilon'(\epsilon)} A_j \quad (61)$$

$$\tilde{a}'_j \sim \frac{-i}{\epsilon'(\epsilon)} \delta_{\Pi j} + \frac{1}{1 + \epsilon'(\epsilon)} A_j, \quad (62)$$

where relative signs matter, while constant factors have been omitted. Keeping ϵ' small but *finite*, we collect the resulting correction terms for $\hat{\mathcal{H}}$, which accordingly deform the QFT:

$$\hat{\mathcal{H}}_{\epsilon'} \equiv \epsilon' e'_0 \left(i\hat{\Pi} \gamma^0 \gamma^j T \gamma_5 C \hat{\psi}(a'_j - \tilde{a}'_j) - \hat{\psi}^\dagger \gamma^0 \gamma^j \hat{\psi} \tilde{a}'_j + \hat{\Pi}^\dagger \gamma^0 \gamma^j \hat{\Pi} a'_j \right). \quad (63)$$

With ϵ' spatially homogeneous, the $F\tilde{F}$ -terms from Eq. (39) are absent.

The second and third terms are of the usual “ $j \cdot A$ ” form. However, the visible current couples to the ghost vector potential and the ghost current couples to the visible vector potential. These lead to *vacuum decay*, thereby lowering the total energy of the system indefinitely. Similarly as the visible-ghost coupling induced by graviton loops, the topic of Ref. [1], the situation here could even be phenomenologically acceptable, if the effective coupling $\epsilon' \epsilon'_0$ is sufficiently small. – Meanwhile, the first term of $\hat{\mathcal{H}}_{\epsilon'}$ couples a transition current of visible and ghost charges to the visible and ghost vector fields. Like the other two, it violates $U(1)_{vis}$ or $U(1)_{gho}$ separately, but leaves the overall local $U(1)$ symmetry intact. Therefore, limits on violation of *charge conservation* in the visible sector could also constrain the size of the effective coupling. – In any case, the present model should not be applied to phenomenology directly. We are rather interested in its new structural features which might be reflected in more realistic theories.

We have seen in Section 3.2 that the first term of $\hat{\mathcal{H}}_{\epsilon'}$ is *not* Hermitian. Concerning the quantum limit, it introduces a *decoherence mechanism*. This may be welcome as necessary ingredient for attempts to solve the measurement problem and, more specifically, the problem of reduction or wave function collapse [23, 31]. Non-unitarity also appears related to the “information loss” deemed necessary by ’t Hooft, in order to base quantum theory on deterministic dynamics [13, 17, 21, 14]. – Presently, the dissipative interaction arises in the deformation of the quantum field theory which relates it to the classical ensemble theory with given symmetries.

3.4 Locality or How to circumvent Bell’s theorem

From $\hat{\mathcal{H}} \equiv \int d^3x \hat{\mathcal{H}}(x)$ we can read off the Hamiltonian density. Explicit calculation then shows:

$$[\hat{\mathcal{H}}(x), \hat{\mathcal{H}}(x')] = 0 \quad , \quad \text{for } x \neq x' \quad . \quad (64)$$

This establishes the locality of the dynamics described by the generalized Hamiltonian, Eqs. (42)–(43), and especially by $\hat{\mathcal{H}} + \hat{\mathcal{H}}_{\epsilon'}$ from Eqs. (57) and (63). – However, this also raises the question how the quantum theory of Section 3.3 could possibly emerge from the generalized classical ensemble theory developed before. It appears to contradict Bell’s theorem which rules out *local* hidden variable theories [23, 31].

Two aspects come into play here. Foremost, our ensemble theory *is nonlocal* with respect to variables of the underlying model – a common feature with other emergent quantum models [13, 16, 18, 22]. In particular, in Eq. (19), the canonical momentum Π_i is traded for the vector potential A'_i via the Fourier transform. We remark that even without interactions the ensemble theory for the gauge field turns into the corresponding free QFT plus ghost copy.

For the fermionic fields, where nothing like an integral transform has been applied, the anticommutativity of the “pseudoclassical” [25, 26] Grassmann variables is sufficient. We recall that the functional Schrödinger equation, Eq. (16), follows entirely from the classical Liouville equation, with added diffusion term $\hat{\mathcal{H}}_{int}$. However, we have found in Section 3.2 that the ensemble theory which smoothly connects to quantum theory lives in a larger Hilbert space than the classical one, concerning the fermionic fields.

We conclude here that our results do not contradict Bell’s theorem.

4 Conclusions

Presently it has been demonstrated that a classical ensemble theory can be deformed into a QFT in agreement with the KS scenario involving energy-parity symmetry [1]. – We have related this deformation to a “varying alpha” coupling in an Abelian $U(1)$ gauge theory [9, 10, 11, 12]. – Founded on a set of physical axioms, the existence of a one-parameter deformation has been

shown which connects the nonrelativistic Schrödinger theory with a classical ensemble theory [32]. Here we provide a relativistic field theory realization of such a deformation.

Our approach has incorporated an element of nonlocality, which is essential for the emerging bosonic variables. It also employs the “pseudoclassical mechanics” based on anticommuting Grassmann variables [25], in order to include classical charged particles with spin which emerge as fermions. Combined with the Hilbert space formalism for the phase space description of a *classical system* [15], this has led, in a particular limit of the coupling, to the corresponding *quantum field theory*. We have discussed necessary dissipative and diffusive interactions mediating between classical and quantum limits, which imply a decoherence mechanism.

A number of open problems and interesting topics for future study arise. – We have pointed out the larger-than-classical Hilbert space related to the realization of fermionic function space and operators. Had we considered charged scalar fields instead, this feature were absent, and a model like scalar QED plus ghost copy results directly from a classical Hermitian ensemble theory. Therefore, it will be interesting to further deconstruct fermions in terms of classical concepts within the present approach. – Regarding more realistic models, an extension to non-Abelian gauge theories seems very important, since nonlinear selfinteractions might obstruct energy-parity and symmetry doubling. – Since we assumed a given deformation parameter, this raises the question whether a selfconsistent model can be built, dealing with gravity and closer in spirit to the KS scenario, which partly motivated the present study [1].

Ending on a speculative note, if a wider range of deterministic quantum models can be constructed, this will likely challenge current ideas about space-time at the Planck scale. Perhaps energy-parity will play a role in this and in solving the old cosmological constant problem.

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